Failure Accommodation in Digital Flight Control Systems by Bayesian Decision Theory

Raymond C. Montgomery* and Alper K. Caglayan†
NASA Langley Research Center, Hampton, Va.

A design method for digital control systems which is optimally tolerant of failures in aircraft sensors is presented. The functions of this system are accomplished with software instead of the popular and costly technique of hardware duplication. The approach taken, based on *M*-ary hypothesis testing, results in a bank of Kalman filters operating in parallel. A moving window of the innovations of each Kalman filter drives a detector that decides the failure state of the system. The detector calculates the likelihood ratio for each hypothesis corresponding to a specific failure state of the system. It also selects the most likely state estimate in the Bayesian sense from the bank of Kalman filters. The system can compensate for hardover as well as increased noise-type failures by computing the likelihood ratios as generalized likelihood ratios. The design method is applied to the design of a fault tolerant control system for a current configuration of the space shuttle orbiter at Mach 5 and 120,000 ft. The failure detection capabilities of the system are demonstrated using a real-time simulation of the system with noisy sensors.

Introduction

THE most striking impact of new technology in aircraft flight control stems from the advent of the modern, high-speed, digital computer. Control concepts previously considered untractable can now be considered because of the flexibility and speed of information processing made available by this new technology. One important new potential that exists is the ability of digital system to reorganize itself to accommodate for failures in sensors and actuators. This reorganization is possible, provided there is enough duplication of function between the actuators or the sensors in a given control system. This paper presents a design method for digital flight control systems that will be optimally tolerant of sensor failures.

Modern control methods allow one to determine the part of the state space of an aircraft that can be dynamically influenced by a given actuator (the controllability subspace) and the part of the state space that a given sensor can produce information about using state estimator theory (observability subspace). Reference 1 provides a good treatment of theoretical considerations involved in determination of these subspaces. Redundancy is provided in either sensors or actuators when there is overlapping of the subspaces of the various sensors or actuators in a given system. For example, consider the longitudinal dynamics of an airplane. If there are three sensors on the aircraft, say an accelerometer, to measure normal acceleration, a pitch-rate gyro, and an elevator position transducer, and if the aircraft state is completely observable from outputs of either sensor, then it is possible, using say a minimum order observer,[2] to estimate the behavior of one sensor based on the output of another one. Redundancy, in that situation, does exist and can be used by cross-checking state estimates obtained by one sensor with those obtained from another one.

Theoretical considerations for determining the absolute level of redundancy that exists in a given system were developed in Ref. 3. Reference 3 also presented a failure detection filter designed to make use of the system redundancy. One limitation of that work was that no consideration of the practical noise environment of the sensors was made and failure detection depended on observing a steady-state bias in an error plane in a state space. For aircraft applications, however, a design process is desired that enables rapid detection of failures during maneuvering transients and accounts for the normal operational noise environment of the aircraft and the control system actuators and sensors.

A design method is presented for resolving both problems in that it accounts for noise in sensors and is capable of determining hardover as well as increased noise-type failures during maneuvering transients. Incorporation of failure detection and recovery into an aircraft control system design is a joint detection, estimation, and control problem. The design method presented here produces a decision for detecting system failures which is optimal in the Bayesian sense. In addition, because of the theoretical development, one is able to account for uncertainty in the aircraft's stability derivatives, mass, inertia, and geometric characteristics. Although the method developed can be applied to both sensor and actuator failures, only sensor failure detection and recovery are considered.

The approach taken here uses M-ary hypothesis testing with generalized likelihood ratios. The elements of this theory were originally developed at the close of World War II for a binary hypothesis testing problem of determining whether a radar return signal respresented a target or not. In that case there are clearly two hypotheses—either there is a target or there is not. Theoretically, one can assign a cost to either failing to detect a real target or creating a false alarm. A performance index can be constructed which expresses the cost of making a decision based on a given radar return. This index can be minimized by selection of threshold points for decision whether or not the return represents a target. Elements of this problem are outlined in Ref. 4, which also contains a brief description of the M-ary hypotheses testing and generalized likelihood ratios. In this paper the set of hypotheses used is, first, that all sensors are functioning properly and, then, $M-1$ further hypotheses stating that the ith sensor group has failed $i=1,2,...,M-1$. In the next section the theory for applying M-ary hypothesis testing to self-reorganizing systems is presented. Then, it is applied to an example aircraft problem.

Sensor Failure Accommodation Using M-ary Hypothesis Testing

Consider the equations of motion of an aircraft to be represented by

Presented as Paper 74-21 at the AIAA 12th Aerospace Sciences Meeting, Washington, D.C., January 30-February 1, 1974; submitted April 12, 1974; revision received March 21. 1975.

Index category: Navigation, Control, and Guidance Theory.

*Aerospace Technologist, Flight Dynamics and Control Division. Member AIAA.

†Research Associate, Virginia Polytechnic Institute and State University, in residence at Langley Research Center.

$$\dot{x} = Ax + Bu + w \tag{1}$$

where x is an n-dimensional state vector, u is an m-dimensional control vector, w is a zero mean Gaussian white noise process with a covariance matrix $W\delta_D(t-\tau)$. In Eq. (1) matrices A and B are determined from the aircraft's stability and control derivatives, its mass and inertia characteristics, and its geometric characteristics. The variable w may, but need not, represent turbulence. It may represent uncertainty in the designer's knowledge of the characteristics of the aircraft. Basically, it can be thought of as representing the error in calculation of $\dot{x}$, given x and u. We will be concerned with the digital control of the plant where the control is constrained to be constant with sampling interval T, that is $u(t) = u\ (kT)$ for $kT \le t < (k+1)T$. By integrating the system differential Eq. (1) over each sampling interval,[4] we get the discrete equations of motion for the aircraft

$$x(k+1) = \Phi x(k) + \Gamma u(k) + w(k) \tag{2}$$

where $x(k) = x(kT)$, $u(k) = u(\mathrm{k}T)$, $\Phi = \Phi(T)$, $\Phi(s) = e^{As}$

$$\Gamma = \int_0^T \Phi(s)\ \mathrm{d}sB$$

and $w(k)$ is a zero mean, white Gaussian sequence with covariance $E\ w(k)\ w'\ (j) = Q\ \delta_{kj}$ where

$$Q = \int_0^T \Phi'(s)\, W\Phi(s)\ \mathrm{d}s \tag{3}$$

Let us assume that the control system has $M-1$ sensor failure modes for each mode

$$y(k) = C_i x(k) + v_i(k) \quad i = 1,2,\ldots,M-1$$

where $v_i(k)$ is a Gaussian white noise sequence where

$$E[v_i(k)] = (0, \bar{m}_i', 0)' \equiv m_i'$$

and

$$E[v_i(k) v_i'(j)] = R_i \delta_{kj}$$

The quantity $\bar{m}_i$ is an unknown (nonrandom) parameter vector.

We shall solve the problem as if m_i were known and then use the maximum likelihood estimate of m_i under the ith hypothesis. This procedure is known as generalized likelihood ratio approach in the communication literature.[5] This approach to failure modeling enables the designer to compensate for hardover failures of arbitary magnitude. Increased sensor noise-type failures can be modeled by appropriate selection of the noise variances R_i.

For the normal unfailed condition we will asume

$$y(k) = C_0 x(k) + v_0(k)$$

where $E[v_0(k)] = 0$ and $E[v_0(k) v_0'(j)] = R_0 \delta_{kj}$. Hence, for a system with three failure modes, as considered in the next section, we have four hypotheses to consider

$$H_0\colon\ y(k) = C_0 x(k) + v_0(k)$$

$$H_1\colon\ y(k) = C_1 x(k) + v_1(k)$$

$$H_2\colon\ y(k) = C_2 x(k) + v_2(k)$$

$$H_3\colon\ y(k) = C_3 x(k) + v_3(k)$$

where C_i ($i = 1, 2, 3$) is C_0 matrix with the rows corresponding to the ith group of sensors replaced by zeros.

We will be concerned with the selection of the most probable hypothesis, based on a finite set of measurements, $Y(K) = \{y(1), y(2), y(2) \ldots y(K)\}$. To do this we construct a Bayesian cost function for the M-ary problem

$$\mathcal{B} = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} P_{H_j} C_{ij} \int_{Z_i} p_{Y|H}(\alpha | H_j)\ \mathrm{d}\alpha \tag{4}$$

subject to

$$\sum_{i=0}^{M-1} P_{H_i} = 1$$

and where the sets Z_i, $i = 0, 1 \ldots M-1$, are disjoint and their union represents the entire observation space. P_{H_i} is the a priori probability of hypothesis H_i being true, C_{ij} is the cost of selecting H_i when H_j is true, and $P_{Y|H}\ (\alpha | H_j)$ is the conditional probability density of the measurement sequence Y given that H_j is true. The symbol $\int_{Z_i}$ implies that the integral is carried over the decision region Z_i in the observation space. Decision regions Z_i are subsets of observation space such that if Y is in Z_i then the hypothesis H_i is to be selected. Note that the integral in Eq. (4) represents nothing more than the probability of making the incorrect decision of selecting hypothesis H_i when H_j is true for $i \neq j$. So the Bayes risk $\mathcal{B}$, represents the sum of probabilities corresponding to different decisions weighted by the a priori probabilities P_{H_i} and the design weights C_{ij}. The problem is to choose the boundaries of decision regions Z_i that will result in minimum Bayes risk. These boundaries are, in effect, switching hypersurfaces for the decision logic in the measurement space.

The minimization of Bayes risk can be performed easily by rewriting the cost function (4) in the form

$$\mathcal{B} = \sum_{i=0}^{M-1} \int_{Z_i} \psi_i(\alpha)\ \mathrm{d}\alpha \tag{5}$$

where

$$\psi_i(\alpha) = \sum_{j=0}^{M-1} P_{H_j} C_{ij} P_{Y|H}(\alpha | H_j) \tag{6}$$

The Bayes risk is minimized by selecting H_i at each point α in the observation space such that $\psi_i(\alpha)$ is the smallest of M possible values of $\psi_i(\alpha)$ $(k = 0, 1 \ldots M-1)$. Hence, the optimal decision regions are

$$Z_i\{\alpha | \psi_i(\alpha) = \min \psi_k(\alpha),\quad 0 \le k \le M-1\} \tag{7}$$

From a computational point of view, it is convenient to introduce a dummy hypothesis H_M with a priori probability $P_{H_M} = 0$ with $H_M\colon\ y(k) = v_0(k)$. Then, an equivalent decision criterion can be given in terms of likelihood ratios, $\Lambda_i(\alpha)$

$$\Lambda_i(\alpha) = P_{Y|H}(\alpha | H_i) / P_{Y|H}(\alpha | H_M) \quad i = 0, 1, \ldots M-1 \tag{8}$$

Dividing each ψ_i in Eq. (6) by the probability density of $Y(K)$ under H_M, we get an equivalent decision criterion in terms of the likelihood ratios

$$\Lambda_i(\alpha) = \sum_{j=0}^{M-1} P_H C_{ij} \Lambda_j(\alpha) \tag{9}$$

Then

$$Z_i = \{\alpha | \lambda_i(\alpha) = \min \lambda_k(\alpha),\ 0 \le k \le M-1\} \tag{10}$$

The advantage of using likelihood ratios is that the boundaries of the decision regions are linear hyperplanes and not general hypersurfaces in the likelihood ratio space $\Lambda_1, \Lambda_2, \ldots,$

Λ_{M-1}. From the chain rule of probability densities and the Gaussian density of the observations, it can be shown[5] that the likelihood ratio for the problem considered is given by

$$\Lambda_i[Y(K)] = \left\{ \prod_{k=1}^{K} \frac{(\det R_0)^{1/2}}{[\det Q_i(k)]^{1/2}} \right\}$$

$$\exp\left\{ -\tfrac{1}{2} \sum_{k=1}^{K} [r_i'(k) Q_i^{-1}(k) r_i(k) - y'(k) R_0^{-1} y(k)] \right\} \quad i=0,1 \ldots M-1 \tag{11}$$

where $r_i(k)$ is the innovation[6] of the measurements under the ith hypothesis given by

$$r_i(k) = y(k) - C_i \hat{x}_i(k|k-1) - m_i(k) \tag{12}$$

with $\hat{x}_i(k|k-1) \equiv E\ [x(k)\,|\,Y(k-1),\ H_i]$. The matrix $Q_i(k)$ in Eq. (11) is given by

$$Q_i(k) = C_i V_{\tilde{x}_i}(k|k-1) C_i' + R_i \tag{13}$$

where $V_{\tilde{x}_i}(k|k-1)$ is the prediction error variance of the estimate of the state $x(k)$ under the ith hypothesis defined by

$$V_{\tilde{x}_i}(k|k-1) = E\{ [x(k) - \hat{x}_i(k|k-1)] [x(k) - \hat{x}_i(k|k-1)]' \,|\, Y(k-1), H_i \}$$

In Eq. (12), the true value of $m_i(k)$ should be used to get the exact likelihood ratio. Since this is not available, we will use the sample mean of $[0, \bar{m}_i(j), 0]'$, $j=1, 2, \ldots k$, that is the maximum likelihood estimate of m_i at the kth instant under the ith hypothesis. That makes Λ_i a generalized likelihood ratio.[5]

To compute $\hat{x}_i(k|k-1)$ and $V_{\tilde{x}_i}(k|k-1)$, M Kalman[7] filters are required. A bank of Kalman filters operting in parallel has been used for parameter adaptive control in Ref. 8. The filter equations are listed as follows for completeness

$$\hat{x}_i(k) = \hat{x}_i(k|k-1) + K_i(k) r_i(k) \tag{14}$$

$$\hat{x}_i(k|k-1) = \Phi \hat{x}_i(k) + \Gamma u(k) \tag{15}$$

where $\hat{x}_i(k)$ is the estimate of the aircraft state under the ith hypothesis defined by

$$\hat{x}_i(k) = E(x(k)\,|\,Y(k), H_i)$$

The filter gain $K_i(k)$ in Eq. (13) can be calculated recursively from the algorithm

$$K_i(k) = V_{\tilde{x}_i}(k|k-1) C_i' Q_i^{-1}(k) \tag{16}$$

where $Q_i(k)$ is given by Eq. (13) and the prediction error variance $V_{\tilde{x}_i}(k|k-1)$ is given by

$$V_{\tilde{x}_i}(k|k-1) = \Phi V_{\tilde{x}_i}(k-1) \Phi' + Q \tag{17}$$

where $V_{\tilde{x}_i}(k)$ is the filter error variance given by

$$V_{\tilde{x}_i}(k) = [I - K_i(k) C_i] V_{\tilde{x}_i}(k|k-1) \tag{18}$$

Note that, because of the special structure of the matrices C_i, the unknown mean m_i does not enter the filter equations. That is, the estimates of the hypothesis conditioned filters will be exact. Thus, for each hypothesis we have a Kalman filter, as previously indicated, that can be used to determine the likelihood ratios, which can, in turn, be used to make the decision as to which hypothesis is most likely. The structure of the system is schematically indicated in Fig. 1.

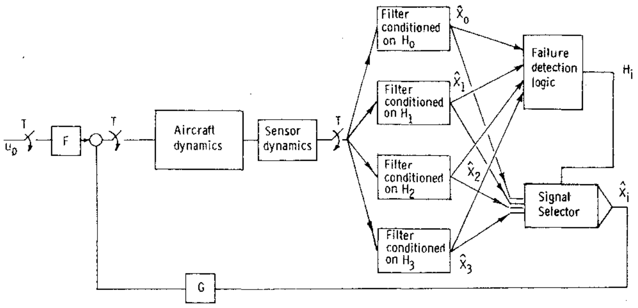

Fig. 1 Fault tolerant control system structure.

Considerable simplification occurs if one considers $C_{ij} = C_{ji} = 1$, $(j \neq 1)$ and $C_{ii} = 0$. Ramifications of this assumption are discussed in the example to follow. Under those conditions the equations for Λ_i may be modified without loss of generality to select the maximum of

$$\{ \ln P_{H_i} - \ln \sum_{j=1}^{K} |Q_i(j)| - \tfrac{1}{2} \sum_{j=1}^{K} r_i'(j) Q_i^{-1}(j) r_i(j), \quad i=0,1,\ldots M-1 \}$$

where K is the total number of measurements used to make the decision. If the steady-state Kalman filter is used, we can select the largest of

$$\{ \ln P_{H_i} - \frac{K}{2} \ln |Q_i| - \tfrac{1}{2} \sum_{j=1}^{K} r_i'(j) Q_i^{-1} r_i(j), \quad i=0,1,\ldots M-1 \}$$

Also, if the a priori probabilities of H_i are equal, without loss of generality, we may take

$$\tau_i[Y(K)] = \frac{K}{2} \ln |Q_i| + \tfrac{1}{2} \sum_{j=1}^{K} r_i'(j) Q_i^{-1} r_i(j) \tag{19}$$

and select the hypothesis H_i corresponding to the smallest τ_i, $i=0,1 \ldots M-1$. The next section demonstrates the application of this method to a practical problem.

Application to Aircraft Flight Control

The theory developed in the previous section has been applied to the design of a control system for one space shuttle orbiter configuration at a Mach number of 5 and an altitude of 120,000 ft. Taking the state to be defined as $x = (p, \phi, r, \beta)'$ and the only effective control $u = \delta_a$ the aircraft equations of motion can be written as

$$\dot{x} = \begin{bmatrix} -0.0580 & 0 & 0.0170 & -5.791 \\ 1.0 & 0 & 0.5773 & 0 \\ -.0029 & 0 & -0.0085 & -0.7438 \\ 0.5 & 0.0055 & -0.8660 & -0.0009 \end{bmatrix} x + \begin{bmatrix} 2.256 \\ 0 \\ 0.0553 \\ 0 \end{bmatrix} u + w$$

Table 1 Evaluation of W

W component	Level of certainty[a]	Error scale
W_1 (p component)	2	0.05 rad/sec
W_2 (ϕ component)	0	1 rad
W_3 (r component)	2	0.01 rad/sec
W_4 (β component)	3	0.001 rad/sec

[a] 0 implies absolute certainty, 1 implies a high level of certainty, 2 implies only moderate certainty, and 3 implies not too sure.

The selection of the variance W for the last equation involves consideration of 1) the uncertainty that we, as designers, feel related to our knowledge of the equations of motion, 2) the relative scales of the variables, and 3) the environment, with regard to turbulence, under which the vehicle must operate. We will only consider the first two items here. Table 1 shows the authors' interpretation of the level of certainty and scale considerations. Concerning the level of certainty, it was felt that the $\dot{\phi}$ equation was well understood since it represents a well-known kinematic relationship. A high level of certainty was assigned to the $\dot{\phi}$ equation. On the other hand, the $\dot{p}$ and $\dot{r}$ equations were felt to be better defined than the $\dot{\beta}$ equation. Turning to scale considerations we have, in effect, equated an error of 1/0.05 in the computation of $\dot{p}$ to one of 1/0.001 in the computation of $\dot{\beta}$. The W matrix selected is constructed from the elements of Table 1 as follows

$$W = \mathrm{diag}\ [2(0.05)^2,\ 0(1)^2,\ 2((1.01)^2,\ 3(0.001)^2]$$

The discretized equations of motion using a zero-order-hold with a sampling interval of 0.1 sec is

$$x(k+1) = \begin{bmatrix} 0.9798 & -0.0002 & 0.0267 & -0.5752 \\ 0.0992 & 1 & 0.0587 & 0.0310 \\ 0.0021 & 0 & 1.002 & -.0740 \\ 0.0497 & 0.0006 & -0.0862 & 0.9887 \end{bmatrix} x(k) + \begin{bmatrix} 0.2240 \\ 0.0139 \\ 0.00536 \\ 0.00538 \end{bmatrix} u(k) + w(k)$$

and the discrete variance matrix for the process $w(k)$ is

$$Q = \begin{bmatrix} 0.4757 & 0.04757 & -0.0066 & 0.0236 \\ 0.04757 & 0.00654 & 0.00100 & 0.00309 \\ -0.006 & 0.00100 & 0.02015 & -0.00185 \\ 0.0236 & 0.00309 & -0.00185 & 0.00211 \end{bmatrix} \times (10)^{-3}$$

which was evaluated using Eq. (3). Note that, because of the sampling, even though the $\dot{\phi}$ equation was considered absolutely certain, uncertainty does result in the $\dot{\phi}$ equation of the discrete model. Also, the components of the plant noise vector are correlated in the discrete model.

For illustration, consider that the vehicle has three sensors: a roll-rate gyro, a yaw-rate gyro, and a sideslip indicator. There will, therefore, be four hypotheses to consider, as follows

$$H_0{:}\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + v_0$$

$$H_1{:}\ y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + v_1$$

$$H_2{:}\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + v_2$$

$$H_3{:}\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + v_3$$

Variances of the measurement error are taken, consistent with current technological capability, to be

$$R_0 = \mathrm{diag}\ [\ (0.05)^2,\ (0.01)^2,\ (0.01)^2\ ]$$

Failure covariances are assumed to be larger than the unfailed ones. (The behavior of the resulting system will, however, be illustrated for both statistical failures—increased variance—and for hardover failures. This capability is a direct result of not assuming a zero-mean measurement error in the failure states.) The values of R_1, R_2, and R_3 used are

$$R_1 = \mathrm{diag}\ (0.025,\ 0.0001,\ 0.0001)$$

$$R_2 = \mathrm{diag}\ (0.0025,\ 0.001,\ 0.0001)$$

$$R_3 = \mathrm{diag}\ (0.0025,\ 0.0001,\ .01)$$

For each hypothesis the state is observable but, given the measurement errors and uncertainties in the vehicle equations of motion, each hypothesis has a different capability of estimating the state of the aircraft. Hence, embedded in the theory is the consideration of the capability of any given sensor group, corresponding to each hypothesis, to estimate the state of the aircraft. This is reflected in the error covariance matrix elements of each hypothesis. As an example, $E[(\hat{p}-p)^2]$ under each hypothesis is indicated as the (1, 1) element of the error covariance matrix and is 0.00075, 0.0015, 0.00082, 0.00087 for H_0, H_1, H_1, and H_3, respectively. As expected, H_0 has the smallest value of $E[(\hat{p}-p)^2]$, indicating that this hypothesis, if true, can produce the best estimate of p. Also indicated, however, is the fact that H_1 produces the worst estimate. Again, this is expected since H_1 corresponds to deletion of roll-rate gyro information.

In this example, The Bayesian risk weights C_{ij} are taken as $C_{ij}=1$ for $i \neq j$ and $C_{ii}=1$. Also, steady-state Kalman filters are used so that Eq. (18) is applicable. For the example here, Eq. (18) becomes (using a memory size of five samples)

$$\tau_0 = -\frac{5}{2}(22.508) + \frac{1}{2}\sum_{k=1}^{5} r_0'(k)$$

$$\begin{bmatrix} 300 & 0.6 & 340 \\ 0.6 & 6350 & 550 \\ 340 & 550 & 3552 \end{bmatrix} r_0(k)$$

$$\tau_1 = -\frac{5}{2}(20.020) + \tfrac{1}{2}\sum_{k=1}^{5} r_1'(k)$$

$$\begin{bmatrix} 40 & 0 & 0 \\ 0 & 6335 & 542 \\ 0 & 542 & 2000 \end{bmatrix} r_1(k)$$

$$\tau_2 = -\frac{5}{2}(20.438) + \tfrac{1}{2}\sum_{k=1}^{5} r_2'(k)$$

$$\begin{bmatrix} 300 & 0 & 340 \\ 0 & 1000 & 0 \\ 340 & 0 & 2893 \end{bmatrix} r_2(k)$$

$$\tau_3 = -\frac{5}{2}(18.897) + \tfrac{1}{2}\sum_{k=1}^{5} r_3'(k)$$

$$\begin{bmatrix} 262 & -52.5 & 0 \\ -52.5 & 6106 & 0 \\ 0 & 0 & 100 \end{bmatrix} r_3(k)$$

During control system operation the scalars τ_i should be using the innovations r_i of the Kalman filter bank stored over the past five samples. Then, the hypothesis corresponding to the minimum τ_i should be selected.

The behavior of the system has been studied using a hybrid computer facility in which the equations of motion of the vehicle were programmed on an analog computer and the control system was mechanized in a digital computer. Figure 2 illustrates the unaugmented step response of the vehicle to an aileron input. This aircraft is a nonminimum phase system indicated by roll reversal. Also, the aircraft possesses a large coupling of the Dutch roll into the aileron response. Digital feedback was employed at a cycle time of 0.1 sec using feedback gains (-4.9, 0.4, 14.5, -6) for (p, $\emptyset$, r, β), respectively, to the aileron. The gains were selected to be constrained to a control system operating with only roll control. Figure 3 shows the response of the closed-loop aircraft to the same pilot step input when state variable feedback (perfect measurement of each state) is employed. Considerable improvement in flying qualities could be obtained if yaw control were available. Figure 4 illustrates the same step response using noisy measurements and accepting H_0. No actuators and sensors have been failed in Fig. 4. In Fig. 5 the responses of the system are indicated for the case where H_2 is true but for each hypothesis being accepted at different times. The failure mode considered in Fig. 5 is an increase in measurement noise. Note that at the start of the record H_0 is selected and produces poor characteristics, as can be seen by comparing the H_0 true portion of the roll-rate trace of Fig. 5 with that of Fig. 4. Had there been no failure, those traces would be almost identical. When H_1 is selected at approximately 5 sec, poor characteristics are still produced. However, when H_2 is selected at approximately 10 sec the system moves to a normal operation, only to return to its poor characteristics when H_3 is selected at approximately 15 sec. This figure illustrates the effect of accepting hypothesis H_i when H_j is true. It indicates the effect of cost selection of the C_{ij} terms in the Bayesian risk function. Figure 6 shows the fault tolerant system in operation when failures of increased noise type are introduced. By looking at the (p, r, β) measurements, it can be seen that the following sensor failure modes have been simulated: $\{H_0, H_1, H_2, H_3\}$. The plot showing the hypothesis accepted indicates the performance of the detector logic. Figure 7 deals with the detection of hardover failures. A hardover failure in the beta sensor has been simulated. Note that, although detection logic is able to detect sensor failures in all cases quite rapidly, the detector logic takes a longer time to reject a failure hypothesis when the

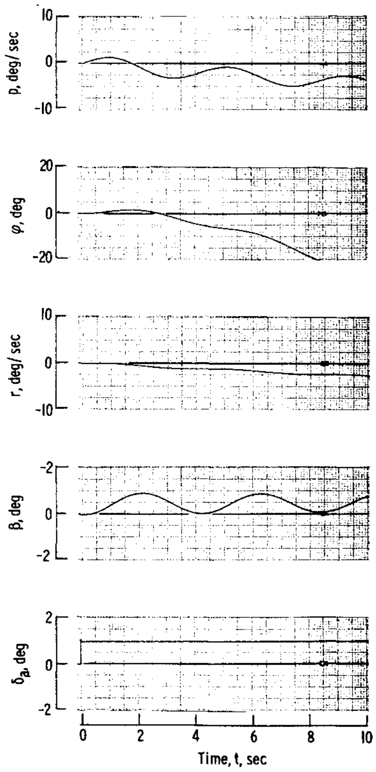

Fig. 2 Response of unaugmented aircraft to aileron step.

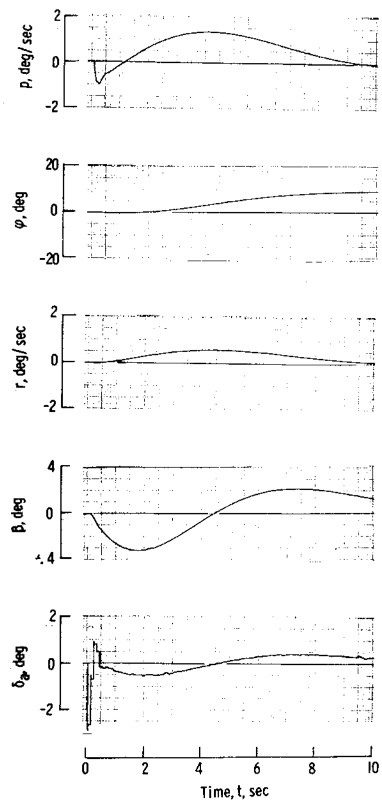

Fig. 3 Response of aircraft to step pilot input when digital state variable feedback (with no measurement noise) is used.

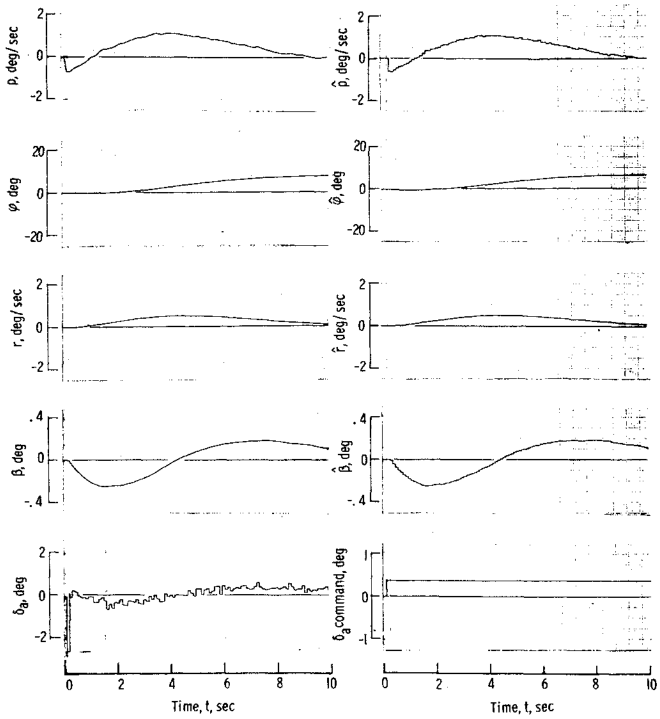

Fig. 4 Response of closed-loop system with noisy measurements under normal operation.

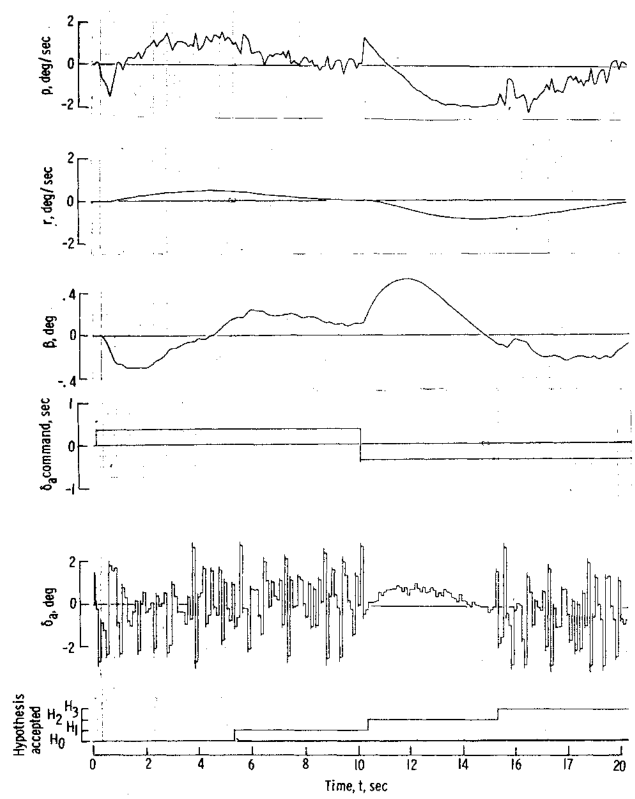

Fig. 5 Response of closed-loop system demonstrating effects of accepting hypothesis H_0, H_2, H_2, when H_2 is true.

system is already in one. Further, note also that only the steady-state Kalman filters are used and overall performance may be improved using time-varying Kalman filters.

Conclusions

A digital fault tolerant control system design that accommodates for aircraft sensor failures has been presented.

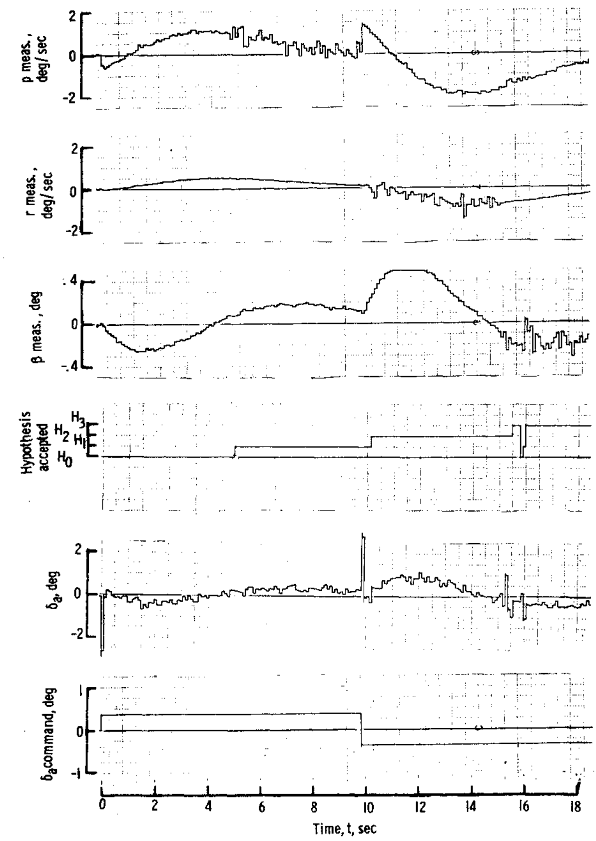

Fig. 6 Operation of fault tolerant system during failures resulting in increased noise.

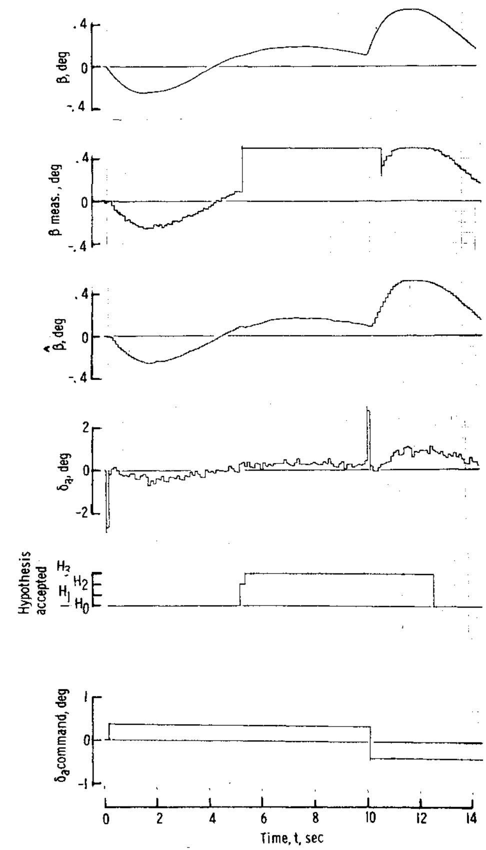

Fig. 7 Operation under saturation hardover failure.

Each sensor failure mode and the normal operation of sensors are modeled as M different hypotheses. Then, using the Bayesian M-ary hypothesis testing approach, a detection logic is developed that results in a bank of M Kalman filters. The decision logic, which uses *M* generalized likelihood ratios, selectes the hypothesis that minimizes the cost of making a wrong decision in the Beyesian sense. The likelihood ratios are calculated from a moving window of the innovations in each of the Kalman filters. The estimate of the state corresponding to the hypothesis selected by the detection logic is used in the control system. The design system is capable of identifying increased noise type and hardover-type sensor failures. These capabilities are demonstrated using a real-time hybrid simulation for a space shuttle vehicle lateral dynamics.

References

[1] Zadeh, L.A., and Desoer, C.A., *Linear Systems Theory,* McGraw-Hill, New York, 1963.

[2] Montgomery, R.C. and Hatch, H.G., Jr., "The Use of Minimum Order State Observers in Digital Flight-Control System," *AIAA Atmospheric Flight Mechanics Conference,* Palo Alto, Calif., Sept. 1972.

[3] Beard, R.V., "Failure Accommodation in Linear Systems through Self-Reorganization," CR-118314, 1971, NASA.

[4] Jazwinski, A.H., *Stochastic Processes and Filtering Theory,* Academic Press, New York, 1970, pp. 195-200.

[5] Sage, A.P. and Melsa, J.L., *Estimation Theory,* McGraw-Hill, New York, 1971, pp. 116-171.

[6] Kailath, T., "An Innovations Approach to Least Equares Estimation, Part I: Linear Filtering in Additive White Noise," *IEEE Transactions on Automatic Control,* Vol. AC-13, Dec. 1968, pp. 646-655.

[7] Kalman, R.E., "New Results in Linear Filtering and Prediction Problems," *Transactions of the ASME, Journal of Basic Engineering,* Vol. 82, March 1960, pp. 35-45.

[8] Desphande, J.G., Upadhyay, T.N., and Lainiotis, D.G., "Adaptive Controls of Linear Stochastic Systems," *Automatica,* Vol. 9, 1973, pp. 107-115.